PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES

Second Edition

Athanasios Papoulis

Polytechnic Institute of New York

McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
Johannesburg London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

EXHIBIT

2

Papoulis

This book was set in Times Roman.
The editor was T. Michael Slaughter;
the production supervisor was Diane Renda.
The cover was designed by Jerry Wilke.
Project supervision was done by Santype International Limited.
Halliday Lithograph Corporation was printer and binder.

PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES

234567890 HALHAL 8987654

ISBN 0-07-048468-6

Library of Congress Cataloging in Publication Data

Papoulis, Athanasios, date
Probability, random variables, and stochastic processes.

(McGraw-Hill series in electrical engineering.
Communications and information theory)
Includes index.
1. Probabilities. 2. Random variables.
3. Stochastic processes. I. Title. II. Series.
QA273.P2 1984 519.2 83-14927
ISBN 0-07-048468-6

Papoulis

Memoryless Systems

A system is called memoryless if its output is given by

$$y(t) = g[x(t)]$$

where $g(x)$ is a function of x. Thus, at a given time $t = t_1$, the output $y(t_1)$ depends only on $x(t_1)$ and not on any other past or future values of $x(t)$.

From the above it follows that the first-order density $f_y(y; t)$ of $y(t)$ can be expressed in terms of the corresponding density $f_x(x; t)$ of $x(t)$ as in Sec. 5-2. We note, in particular, that

$$E\{y(t)\} = \int_{-\infty}^{\infty} g(x) f_x(x; t)\, dx$$

Similarly, since $y(t_1) = g[x(t_1)]$ and $y(t_2) = g[x(t_2)]$, the second-order density $f_y(y_1, y_2; t_1, t_2)$ of $y(t)$ can be determined in terms of the corresponding density $f_x(x_1, x_2; t_1, t_2)$ of $x(t)$ as in Sec. 6-3. In particular

$$E\{y(t_1)y(t_2)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2\, g(x_1) g(x_2) f_x(x_1, x_2; t_1, t_2)\, dx_1\, dx_2$$

The nth order density $f_y(y_1, \ldots, y_n; t_1, \ldots, t_n)$ of $y(t)$ can be determined from the corresponding density of $x(t)$ as in (8-8) where the underlying transformation is the system

$$y(t_1) = g[x(t_1)], \ldots, y(t_n) = g[x(t_n)] \tag{9-86}$$

Stationarity Suppose that the input to a memoryless system is an SSS process $x(t)$. We shall show that the resulting output $y(t)$ is also SSS.

PROOF To determine the nth order density of $y(t)$, we solve the system

$$g(x_1) = y_1, \ldots, g(x_n) = y_n \tag{9-87}$$

If this system has a unique solution, then [see (8-8)]

$$f_y(y_1, \ldots, y_n; t_1, \ldots, t_n) = \frac{f_x(x_1, \ldots, x_n; t_1, \ldots, t_n)}{|g'(x_1) \cdots g'(x_n)|} \tag{9-88}$$

From the stationarity of $x(t)$ it follows that the numerator in (9-88) is invariant to a shift of the time origin. And since the denominator does not depend on t, we conclude that the left side does not change if t_i is replaced by $t_i + c$. Hence, $y(t)$ is SSS. We can similarly show that this is true even if (9-87) has more than one solution.

Notes 1. If $x(t)$ is stationary of order N, then $y(t)$ is stationary of order N.

2. If $x(t)$ is stationary in an interval, then $y(t)$ is stationary in the same interval.

3. If $x(t)$ is WSS stationary, then $y(t)$ might not be stationary in any sense.